

Math 115

Spring 2018

Lecture 28

$$? a^2 + b^2 = c^2 ?$$

$$y = mx + b \quad ? \quad d = rt$$

Review:

① Simplify: $\frac{x^2 - 4}{2x^2 - 3x - 2} = \frac{(x+2)(\cancel{x-2})}{(2x+1)(\cancel{x-2})}$

 $= \frac{x+2}{2x+1}$

② Find all excluded values: $\frac{3x-5}{3x^2+5x-8}$

Denom = 0
Solve

$$3x^2 + 5x - 8 = 0$$

$$ax^2 + bx + c = 0$$

$a=3, b=5, c=-8$

$$b^2 - 4ac = (5)^2 - 4(3)(-8) = 25 + 96 = 121$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{121}}{2(3)}$$



$$x = \frac{-5+11}{6}, x = \frac{-5-11}{6}$$



$$x = \frac{6}{6}$$

$$\boxed{x=1}$$

$$x = \frac{-16}{6}$$

$$\boxed{x = -\frac{8}{3}}$$

$$x = \frac{-5 \pm 11}{6}$$

E.V.: $-\frac{8}{3}, 1$

Simplify:

$$\frac{2x^2 + x - 6}{2x^2 - 9x + 9} \cdot \frac{x^2 - 2x - 3}{x^2 - 1}$$

$$= \frac{\cancel{(2x-3)}(x+2)}{\cancel{(2x-3)}\cancel{(x-3)}} \cdot \frac{\cancel{(x+1)}\cancel{(x-3)}}{\cancel{(x+1)}(x-1)}$$

$$= \boxed{\frac{x+2}{x-1}}$$

Simplify:

$$\frac{3x^2 - x}{6x^2 + 15x} \div \frac{6x^2 + x - 1}{2x^2 - 5x - 25}$$

$$= \frac{3x^2 - x}{6x^2 + 15x} \cdot \frac{2x^2 - 5x - 25}{6x^2 + x - 1}$$

$$= \frac{\cancel{x}(3\cancel{x}-1)}{3\cancel{x}(\cancel{2x}+5)} \cdot \frac{(\cancel{2x}+5)(x-5)}{(2x+1)\cancel{(3x-1)}}$$

$$= \boxed{\frac{x-5}{3(2x+1)}}$$

Simplify

$$\frac{x^2 - 4x}{x^2 + 3x + 2} + \frac{5x}{x^2 + 3x + 2}$$

$$= \frac{x^2 - 4x + 5x}{x^2 + 3x + 2} = \frac{x^2 + x}{x^2 + 3x + 2}$$

$$= \frac{x(x+1)}{(x+2)(x+1)} = \boxed{\frac{x}{x+2}}$$

Simplify

$$\frac{x}{x^2 - 4} - \frac{3}{x^2 - x - 2}$$

$$= \frac{x(x+1)}{(x+2)(x-2)(x+1)} - \frac{3(x+2)}{(x-2)(x+1)(x+2)}$$

$$= \frac{x(x+1) - 3(x+2)}{\underbrace{(x+2)(x-2)(x+1)}_{\text{LCD}}} = \frac{x^2 + x - 3x - 6}{(x+2)(x-2)(x+1)}$$

$$= \boxed{\frac{x^2 - 2x - 6}{(x+2)(x-2)(x+1)}}$$

Simplify:

$$\begin{aligned}
 & \frac{9}{x^2 + 9x + 18} - \frac{4}{x+6} - \frac{3}{x+3} \\
 &= \frac{9}{(x+3)(x+6)} - \frac{4(x+3)}{(x+6)(x+3)} - \frac{3(x+6)}{(x+3)(x+6)} \\
 &= \frac{9 - 4(x+3) - 3(x+6)}{(x+3)(x+6)} = \frac{9 - 4x - 12 - 3x - 18}{(x+3)(x+6)} \\
 & \quad \text{LCD} \\
 &= \frac{-7x - 21}{(x+3)(x+6)} \\
 &= \frac{-7(\cancel{x+3})}{(\cancel{x+3})(x+6)} \\
 &= \boxed{\frac{-7}{x+6}}
 \end{aligned}$$

Simplify:

$$\begin{aligned}
 & \frac{x}{x-2} + \frac{3}{x+2} - \frac{8}{x^2 - 4} \\
 &= \frac{x(x+2)}{(x-2)(x+2)} + \frac{3(x-2)}{(x+2)(x-2)} - \frac{8}{(x-2)(x+2)} \\
 &= \frac{x(x+2) + 3(x-2) - 8}{(x-2)(x+2)} = \frac{x^2 + 2x + 3x - 6 - 8}{(x-2)(x+2)} = \frac{x^2 + 5x - 14}{(x-2)(x+2)} \\
 &= \frac{(x+7)(\cancel{x-2})}{(\cancel{x-2})(x+2)} = \boxed{\frac{x+7}{x+2}}
 \end{aligned}$$

Complex Fraction: It is a fraction that contains at least one fraction.

$$\frac{4 - \frac{2}{3}}{5}, \quad \frac{\frac{3}{5} - \frac{1}{2}}{\frac{7}{10}}, \quad \frac{x - \frac{4}{x}}{1 - \frac{2}{x}}$$

$$\frac{\frac{x+5}{x-2} - \frac{x}{x+6}}{\frac{8}{x^2+4x-12}}$$

To Reduce/Simplify Complex Fraction,

① Find LCD of all denominators

② Multiply everything by LCD.

③ Simplify, Simplify, and Simplify

Simplify

$$\frac{3 - \frac{1}{2}}{\frac{3}{4}} \quad \text{LCD} = 4$$

$$= \frac{4 \cdot 3 - \cancel{4} \cdot \frac{1}{\cancel{2}}}{\cancel{4} \cdot \frac{3}{\cancel{4}}} = \frac{12 - 2}{3} = \boxed{\frac{10}{3}}$$

Simplify

$$\frac{x - \frac{4}{x}}{1 - \frac{2}{x}} = \frac{x \cdot x - x \cdot \frac{4}{x}}{x \cdot 1 - x \cdot \frac{2}{x}} = \frac{x^2 - 4}{x - 2}$$

LCD = x

$$= \frac{(x+2)(\cancel{x-2})}{\cancel{x-2}} = \boxed{x+2}$$

Simplify :

$$\frac{1 + \frac{4}{x} - \frac{5}{x^2}}{1 - \frac{25}{x^2}} = \frac{x^2 \cdot 1 + x^2 \cdot \frac{4}{x} - x^2 \cdot \frac{5}{x^2}}{1 \cdot x^2 - x^2 \cdot \frac{25}{x^2}}$$

LCD = x^2

$$= \frac{x^2 + 4x - 5}{x^2 - 25}$$

$$= \frac{(\cancel{x+5})(x-1)}{(\cancel{x+5})(x-5)}$$

$$= \boxed{\frac{x-1}{x-5}}$$

Simplify: $\frac{3}{x+2} - 4$

$$\frac{\frac{3}{x+2} - 4}{\frac{2}{x+2} + 1} = \frac{\cancel{(x+2)} \cdot \frac{3}{\cancel{x+2}} - (x+2) \cdot 4}{\cancel{(x+2)} \cdot \frac{2}{\cancel{x+2}} + (x+2) \cdot 1}$$

LCD = (x+2)

$$= \frac{3 - 4(x+2)}{2 + x+2}$$

$$= \frac{3 - 4x - 8}{x+4} = \boxed{\frac{-4x-5}{x+4}}$$

Simplify: $6 + \frac{3}{x}$

$$\frac{6 + \frac{3}{x}}{\frac{x}{4} + \frac{1}{8}} = \frac{8x \cdot 6 + \cancel{8x} \cdot \frac{3}{\cancel{x}}}{\cancel{8x} \cdot \frac{x}{\cancel{4}} + \cancel{8x} \cdot \frac{1}{\cancel{8}}}$$

LCD = 8x

$$= \frac{48x + 24}{2x^2 + x}$$

$$= \frac{24 \cancel{(2x+1)}}{x \cancel{(2x+1)}}$$

$$= \boxed{\frac{24}{x}}$$

Solving Rational Equations:

- ① Find LCD & all excluded values
- ② Multiply everything by LCD to clear all fractions.
- ③ Solve the new equation.
- ④ Only keep those answers that are not excluded values.

Solve

$$\frac{x}{2} - \frac{x-1}{3} = 1$$

LCD = 6, E.V.: None

$$\overset{3}{\cancel{6}} \cdot \frac{x}{\cancel{2}} - \overset{2}{\cancel{6}} \cdot \frac{x-1}{\cancel{3}} = 6 \cdot 1$$

$$3x - 2(x-1) = 6$$

$$3x - 2x + 2 = 6$$

$$x + 2 = 6$$

$$\boxed{x=4}$$

$\{4\}$

Solve $\frac{3}{4}x = \frac{1}{2} + \frac{2}{3}x \Rightarrow \frac{3x}{4} = \frac{1}{2} + \frac{2x}{3}$

LCD=12 E.V.: None

$$\cancel{12}^3 \cdot \frac{3x}{\cancel{4}} = \cancel{12}^6 \cdot \frac{1}{\cancel{2}} + \cancel{12}^4 \cdot \frac{2x}{\cancel{3}}$$

$$9x = 6 + 8x$$

$$9x - 8x = 6$$

$$\boxed{x=6}$$

$\{6\}$

Solve $\frac{x}{x-2} - 2 = \frac{2}{x-2} \rightarrow x-2=0$
 $x=2$

LCD = $x-2$

E.V.: 2

$$\cancel{(x-2)} \cdot \frac{x}{\cancel{x-2}} - (x-2) \cdot 2 = \cancel{(x-2)} \cdot \frac{2}{\cancel{x-2}}$$

$$x - 2(x-2) = 2$$

$$x - 2x + 4 = 2$$

$$-x + 4 = 2$$

$$-x = 2 - 4$$

$$\rightarrow -x = -2$$

$$\cancel{x=2}$$

\emptyset

Solve

$$\frac{4}{x+4} + \frac{2}{x-4} = \frac{2x}{x^2-16}$$

$$\text{LCD} = (x+4)(x-4) \quad \text{E.V.: } \pm 4$$

$$\cancel{(x+4)(x-4)} \cdot \frac{4}{\cancel{x+4}} + \cancel{(x+4)(x-4)} \cdot \frac{2}{\cancel{x-4}} = \cancel{(x+4)(x-4)} \cdot \frac{2x}{\cancel{x^2-16}}$$

$$4(x-4) + 2(x+4) = 2x \rightarrow 6x - 2x = 8$$

$$4x - 16 + 2x + 8 = 2x$$

$$6x - 8 = 2x$$

$$4x = 8$$

$$\boxed{x=2} \quad \{2\}$$

Solve

$$\boxed{\frac{x}{x-4}} + \boxed{\frac{1}{x+4}} = \boxed{\frac{-8}{x^2-16}}$$

$$\text{LCD} = (x+4)(x-4), \quad \text{E.V.: } \pm 4$$

$$x(x+4) + 1(x-4) = -8$$

$$x^2 + 4x + x - 4 + 8 = 0$$

$$x^2 + 5x + 4 = 0$$

$$(x+4)(x+1) = 0$$

$$\cancel{x = -4} \quad x = -1$$

$$\{-1\}$$

Solve:

$$\frac{x}{x^2+x-2} + \frac{x}{x^2-1} = \frac{x}{x^2+3x+2}$$

$$\boxed{\frac{x}{(x-1)(x+2)}} + \boxed{\frac{x}{(x+1)(x-1)}} = \boxed{\frac{x}{(x+2)(x+1)}}$$

$$L.C.D = (x-1)(x+2)(x+1)$$

$$E.N.: 1, -2, -1$$

$$x(x+1) + x(x+2) = x(x-1)$$

$$\cancel{x^2} + x + x^2 + 2x = \cancel{x^2} - x$$

$$x^2 + 3x + x = 0$$

$$x^2 + 4x = 0$$

$$x(x+4) = 0$$

By Z.F.P.,

$$\boxed{x=0} \quad x+4=0$$

$$\boxed{x=-4}$$

$$\boxed{\{-4, 0\}}$$